Q.P. Code: 20HS0845 **R20**

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SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR
(AUTONOMOUS)
B.Tech II Year I Semester Regular Examinations May-2022
MATHEMATICAL AND STATISTICAL METHODS
(Common to CSM & CIC)

Time: 3 hours Max. Marks: 60

(Answer all Five Units **5 x 12 = 60** Marks)

UNIT-I

1 a Using the formula for $\pi(n)$ find the number of primes ≤ 100. L2 6M

b For every positive integer n, prove that $7^n - 3^n$ is divisible by 4. L3 6M

OR

2 State and prove fundamental theorem of arithmetic L5 12M

UNIT-II

3 a Find $\phi(200)$, $\sigma(200)$ and $\tau(200)$. L3 6M

b Compute the least residue of $2^{340} \pmod{341}$. L2 6M

OR

4 Solve the system of congruence $x \equiv 3 \pmod{10}$, $x \equiv 8 \pmod{15}$, $x \equiv 5 \pmod{84}$ L6 12M

UNIT-III

5 a Show that the sample variance is a consistent estimator of the population variance σ^2. L3 6M

b Define the following estimators and give an example for each (i) Point estimation (ii) Unbiased estimator (iii) Consistent estimator L3 6M

OR

6 a Let $x_1, x_2, \ldots. x_n$ denote random sample of size n from a uniform population with probability density function $f(x,\theta) = 1;\ \theta - 1/2 \leq x \leq \theta + 1/2,\ -\infty < \theta < \infty$. Obtain Maximum likely estimation for θ. L4 6M

b Find the maximum likelihood estimation of θ in $f(x,\theta) = (1+\theta)\, x^{\theta},\ 0 < x < 1$ based on an independent sample of size n. Examine whether this estimate is sufficient for θ. L5 6M

UNIT-IV

7 a The transition probability matrix of a Markov chain $\{x_n\}$, n=1, 2, 3,...having three states, 1,2 and 3is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.1,\ 0.2,\ 0.1)$. Find $(i)\ P(X_2 = 3,\ X_1 = 3,\ X_0 = 2)$ $(ii)\ P(X_2 = 3)$ $(iii)\ P(X_2 = 2,\ X_2 = 3,\ X_1 = 3,\ X_0 = 2)$ L5 6M

b Suppose a communication system transmits the digits 0 and 1 through many stages. At each state the probability that the same digit will be received by the next stage as transmitted, is 0.75. What is the probability that a 0 is entered at the first stage is received as a 0 in the 5th stage? L3 6M

OR

8	There are two boxes, box I contains 2 white balls and box II contains 3 red balls. At each step of the process, a ball is selected from each box and the 2 balls are Interchanged. Thus, box I always contains 2 balls and box II always contains 3 balls. The states of the system represent the number of red balls in box I after the interchange. Find (i) the transition matrix of the system (ii) the probability that there are 2 red balls in the box I after 3 steps and (iii) the probability that, in the long run there are 2 red balls in box I.	L4	12M

UNIT-V

9	Satyam info way has two persons for its browsing Centre. If the service time for each client is exponential with mean 4 minutes, and if people arrive in a Poisson fashion at the rate of 10 an hour. Then calculate the (i) Probability of having to wait for service (ii) Expected percentage of idle time for each girl (iii) If a client has to wait, what is the expected length of his waiting time?	L5	12M
	OR		
10	A Self-service canteen employee's one cashier at its counter 8 customers arrives per every 10 minutes on an average. The cashier can serve on average one per minute. Assuming that the arrivals are Poisson and the service time distribution is exponential, determine (i) The average number of customers in the system. (ii) The average queue length (iii) Average time a customer spends in the system. (iv) Average waiting time of each customer.	L5	12M

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